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Sinusoidal Variation in Biogas Production from Anaerobic Reactors Operating under Diurnally Cyclic Environmental Temperature

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Abstract— An earlier study conducted by the author had shown that biogas production from anaerobic reactors operated under a diurnally cyclic temperature environment follows a cyclic pattern. In this study, data generated in the said previous study was subjected to spectral analysis to determine whether biogas production under such an environment can be described by a Fourier function. Results showed that gas production from a healthy digester follows a sinusoidal pattern with a dominant frequency which lies at about 1.22 x 10^{-5} Hz corresponding to a period of 24 hours. Where the operating conditions (loading rate, pH, etc) are not favourable, the production follows a sinusoidal pattern which may be embedded in some harmonics and noise.

Index Terms— Anaerobic reactors, Biogas production, Fourier function, Spectral analysis, Diurnally cyclic temperature.

1 Introduction

Temperature is considered to be one of the most important environmental parameters affecting the performance of anaerobic reactors. Diurnally cyclic variation in slurry temperature is a common phenomenon in reactors operated under ambient conditions. A number of studies have been conducted to determine the effects of diurnally cyclic environmental temperature on anaerobic reactor performance [1], [2], [3]. These studies showed that gas production as well as some other operating indices followed a diurnally cyclic pattern with some lag relative to the diurnally cyclic environmental temperature. In this study, the gas production data of Echiegu [1] was subjected to spectral analysis to determine whether gas production under the above operating condition can be described by a perfectly sinusoidally periodic function.

2 Literature Review

Deterministic data is one which can be described by an explicit mathematical relationship; a non-deterministic data cannot. Deterministic data can be classified as periodic or non periodic. Periodic data is one for which the following relationship holds [4], [5]:

$$y(t) = \mathrm{f}(t + T_p) \quad (1)$$

where $y(t)$ is the value of the variable at time t in seconds and T_p is the period in seconds.

Many periodic data can be described in form of a sine or cosine wave. Such a data is said to be sinusoidal. Typically, a sinusoidal data can be represented by a time dependent function of the form shown in equation (2) below:

$$y(t) = A(\cos\omega_o t + \phi) \quad (2)$$

where A is the amplitude, ω_o is the angular frequency (rad/s) and ϕ is the phase angle (rad).

The angular frequency can be expressed as follows:

$$\omega_o = 2\pi f \quad (3)$$

where f is the frequency in Hz = $^1/_{T_P}$.

By invoking trigonometric relationships, equation (2) can be expressed as:

$$y(t) = a_1\cos(\omega_o t) + b_1\sin(\omega_o t) \quad (4)$$

Where

$$a_1 = A\cos\phi \quad (5)$$

And

$$b_1 = A\sin\phi \quad (6)$$

Many real life periodic data are not sinusoidal. They are rather complex in nature and can be described as follows:

$$y(t) = \mathrm{f}(t + nT_p) \quad (7)$$

where n is harmonic number. More generally, arbitrary periodic functions can be represented by an infinite series of sinusoids of harmonically related frequencies such as Fourier series. A Fourier series representation of a complex periodic data can be written as:

$$y(t) = \mathrm{Y_o} + \sum_{\mathrm{n=1}}^{\infty}[a_n\cos(n\omega_o t) + b_n\sin(n\omega_o t)] \quad (8)$$

where Y_o is the mean value of the periodic function. The terms Y_o, a_n and b_n are defined as follows:

$$Y_o = \frac{1}{T_P}\int_0^{T_P} y(t)dt \quad (9)$$

$$a_n = \frac{2}{T_P}\int_0^{T_P} y(t)\cos(n\omega_o t)dt \quad n = 0,1,2,\ldots.. \quad (10)$$

$$b_n = \frac{2}{T_P}\int_0^{T_P} y(t)\sin(n\omega_o t)dt \quad n = 0,1,2,\ldots.. \quad (11)$$

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For equally spaced data with N data points, equations (9) to (11) can be written respectively as follows:

$$Y_o = \sum_{i=1}^{N} \frac{y_i(t)}{N} \quad (12)$$

$$a_n = \frac{2}{N} \sum_{i=1}^{N} [y(t)\cos(n\,\omega_o t)] \quad n = 0,1,2, \ldots.. \quad (13)$$

$$b_n \tau = \frac{2}{N} \sum_{i=1}^{N} [y(t)\sin(n\,\omega_o t)] \quad n = 0,1,2, \ldots.. \quad (14)$$

Equation (8) indicates that a periodic data can be represented by a static component Y_o and an infinite number of sinusoidal components with a fundamental frequency ω_o and harmonic whose frequencies are constant multiples of the fundamental frequency. In periodic data, the sequence repeats, although the repetition may not be exact. To measure the degree of similarity between two successive positions, the sequence would have to be compared with itself at two successive positions. This is achieved by calculating autocorrelation function. Autocorrelation is defined as the linear correlation between a time series at time (t) and the same series at a later time ($t + \tau$) [4]. Mathematically,

$$R_{xx}(t, t+\tau) = \lim_{n\to\infty} \{\frac{1}{n} \sum_{i=1}^{n} [y_i(t) y_i(t\ +\ \tau)]\} \quad n = 0,1,2 \ldots (15)$$

where $R_{xx}(t, t+\tau)$ is the autocorrelation coefficient and τ is the time interval in seconds.

Lag is the amount of offset between two successive series being compared. If each observation in a segment of time series is numbered from 1 to i, the autocorrelation has zero lag when element 1, 2, 3, ...i is compared to elements 1, 2, 3, …i of another segment of the series. When the correlation coefficient of elements 1, 2, 3 …i-1 of one segment of the series are compared to elements 2, 3, 4, …i of another segment of the same series, the autocorrelation has a lag of 1. For a perfectly periodic data such as sine wave, the autocorrelation at zero lag equals 1. A typical autocorrelation will start from +1 at zero lag, decrease to a negative value greater than or equal to -1 and then rise again. For a perfectly periodic data, the autocorrelation coefficient will vary from +1 to -1.

3 Methodology

The effect of diurnally cyclic temperature on the performance of anaerobic digester operated on screened dairy manure was earlier investigated by the author [1]. Two diurnally cyclic temperature regimes (15 – 25°C and 20 – 40°C), two influent total soilds (TS) concentrations (6.4 % and 3.5 % TS) and four levels of hydraulic retention time (HRT) (25, 20, 15 and 10 days) were used in the study. Typical result of the study shown in Figures 1 suggested that gas production under such operating condition followed a diurnally cyclic pattern. To investigate whether the gas production from anaerobic reactor

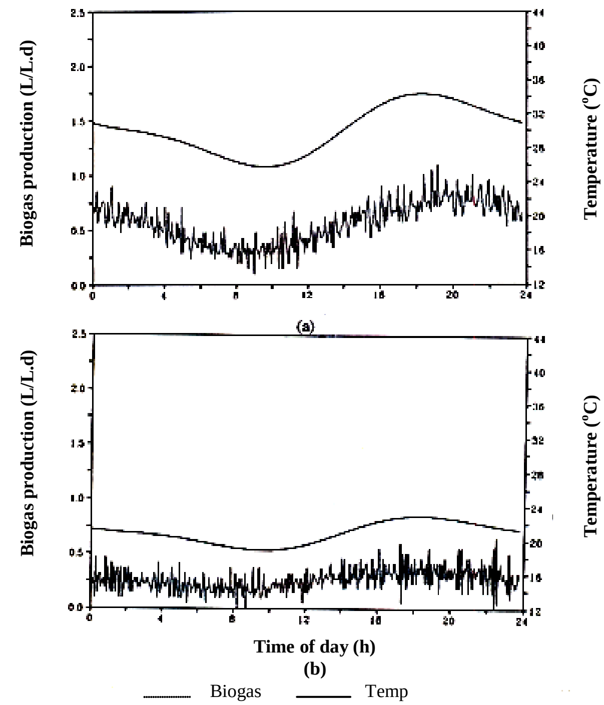

Figure 1. Typical variation in biogas production rate under diurnally cyclic environment. (HRT of 25 days, TS 6.4 % temperature cycle (a) 20 – 40°C and (b) 15 – 25 °C) [1]

under diurnally cyclic environmental temperature can be described as perfectly periodic, the gas production data of Echiegu [1] was subjected to spectral analysis. Computation of autocorrelation coefficients were accomplished by means of Blackman-Turkey procedure [5]. The data was first transformed by removing the means and any trend that existed on the data. The data was further decomposed into its harmonics constituents by using Fourier analysis. Details of the computational procedure and algorithms are given in Marple [5].

4 Results and Discussions

Typical results of the autocorrelation and Fourier analysis are shown in Figure 2 to 5. Figure 2(a) was obtained at 25 days HRT using manure of 6.4 % TS content and the temperature cycle of 20 – 40 °C, while Figure 2(b) was obtained using similar HRT and manure TS content but a temperature cycle of 15 – 25 °C.

From the correlogram, it is seen that the diurnal biogas production rate predominantly followed a pattern similar to a sine wave. This pattern was observed for all HRTs using the manure of 6.4 % TS content and at the operating temperature cycle of 20 – 40°C. It was also observed at 25, 20 and 15 days HRT using the manure of 3.5 % TS at the same operating temperature of 20 – 40°C. However, at 15 – 25°C temperature cycle, the sinusoidal pattern was only followed at 25 day HRT with the influent TS of 6.4 %.

Other typical types of observation revealed by the auto-

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correlation analysis are shown in Figure 3 which was obtained

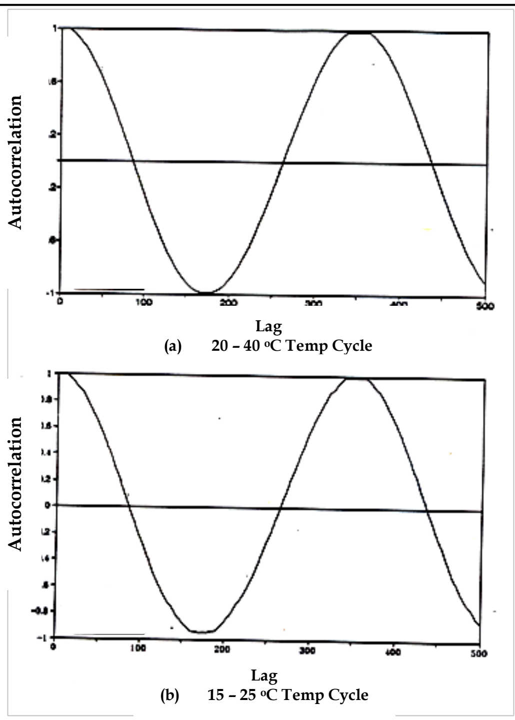

Figure 2. Typical correlogram of the diurnal biogas production rate. (HRT of 25 days, TS 6.4 % temperature cycle (a) 20 – 40°C and (b) 15 – 25 °C)

at 10 day HRT using manure of 3.5 % TS content.

From Figure 3, it can be concluded that under some operating conditions (low HRT, low influent TS Concentration, etc), the biogas production rate follows a sine wave sandwiched in some harmonics and noise.

Various variants of Figure 3 were obtained at all retention times under the operating temperature cycle of 15 – 25 °C using the manure of 3.5 % TS content. They were also recorded at the retention times of 10 to 20 days using the manure of 6.4 % TS at the operating temperature cycle of 20 – 40 °C. Furthermore, a similar correlogram was obtained at the retention time of 10 days using the manure of 3.5 % TS concentration at the operating temperature cycle 20 – 40 °C. These results are summarized in Table 1.

Generally, sinusoidal periodic variations in biogas production rates were obtained at almost all loading rates under the 20 – 40 °C temperature cycle. This is to be expected since the temperature of the reactor used in the study was designed to vary in a diurnally cyclic (sinusoidal) manner and microbial activity and gas production increases with increase in temperature. Sinusoidal variations were not obtained under most of the loading rates at the temperature cycle of 15 – 25 °C. This may have been due to fact that the amplitude of diurnal fluctuation in reactor temperature was relatively so small that it was impossible to detect a true sine wave. At the higher loading rate, pure sinusoidal variation was not obtained probably because of the higher frequency of the feeding cycle (i.e. shorter feeding interval) as most of the evolved gases are scavenged from the headspace of the reactors and measured during the feeding process.

TABLE 1
SUMMARY OF AUTOCORRELATION ANALYSIS

Temperature Cycle (°C)	**Solids Content (%)**	**HRT (days)**	**Sinusiodally Periodic**
20 – 40	6.4	25	Yes
		20	Yes
		15	Yes
		10	Yes
	3.5	25	Yes
		20	Yes
		15	Yes
		10	No
15 – 25	6.4	25	Yes
		20	No
		15	No
		10	No
	3.5	25	No
		20	No
		15	No
		10	No

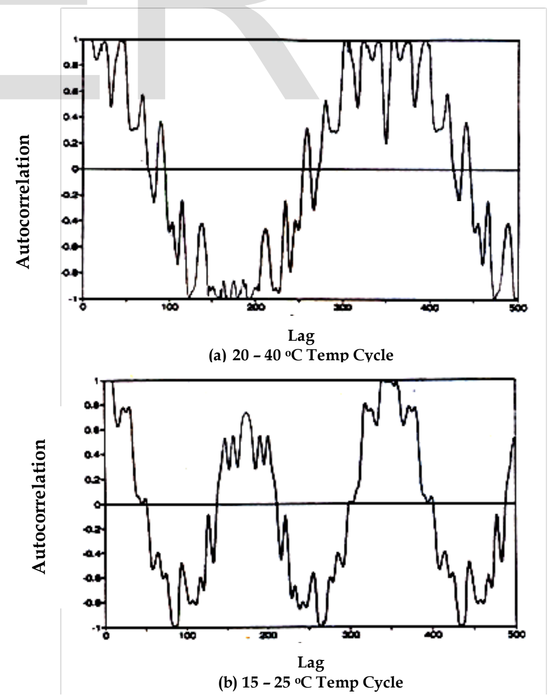

Figure 3. Other typical correlogram of the diurnal biogas production rate. (HRT of 10 days, TS 3.5 % temperature cycle (a) 20 – 40°C and (b) 15 – 25 °C)

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The result of the fast Fourier analysis on the diurnal biogas production data are shown in Figures 4 and 5. These were obtained under the same condition as for Figures 2 and 3 respectively. For the operating conditions where diurnal biogas production cycle followed true sinusoidal relationships, highest Fourier amplitude appears to occurs at a frequency of about 1.22 x 10^{-5} Hz. Thus, the dominant frequency of such gas production cycle lies at about 1.22 x 10^{-5} Hz. This frequency roughly corresponds to a period of 24 hours. Under the condition when the biogas production did not follow a true sinusoidal cycle, the dominant frequency, indicated by the highest Fourier amplitude appears to lie between 0.0013 and 0.0014. Other peaks can be detected.

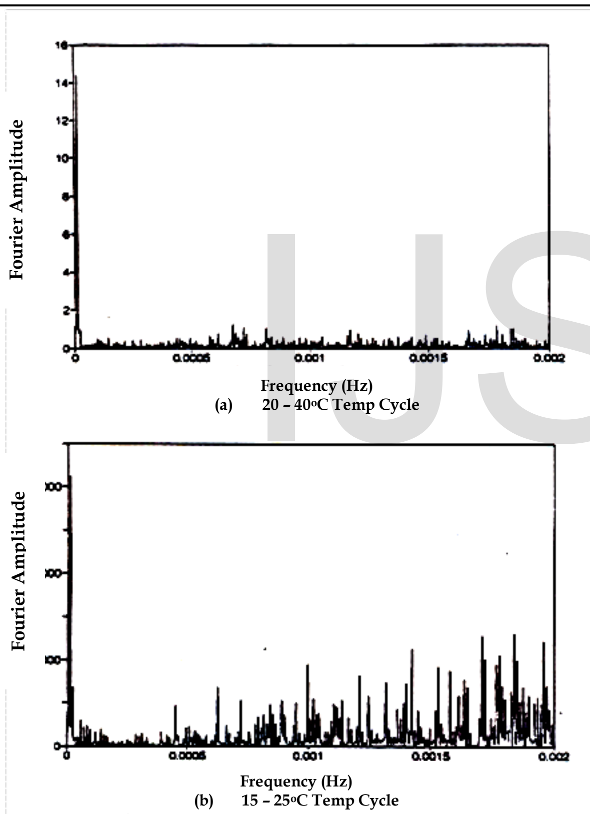

Figure 4. Typical discrete Fourier transform of the data on biogas production rate. (HRT of 25 days, TS 6.4 % temperature cycle (a) 20 – 40°C and (b) 15 – 25 °C)

CONCLUSION

Biogas production from a healthy anaerobic reactor operating under a diurnally cyclic temperature environment follow a sinusoidal pattern with a dominant frequency of 1.22 x 10^{-5} Hz. With the dominant frequency determined, a regression equation based on Fourier series can be written to describe the diurnal variation in biogas production under diurnally cyclic temperature environment. The equation will be of the form:

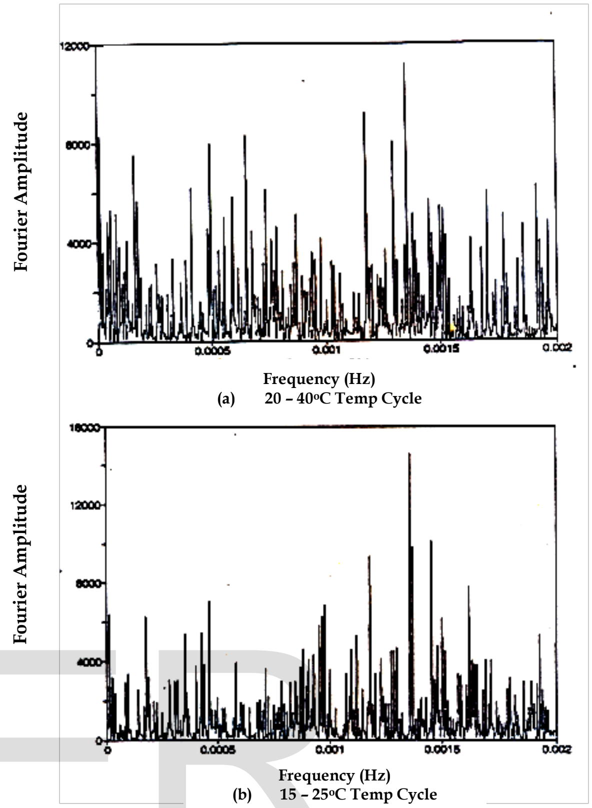

Figure 5. Other typical discrete Fourier transform of the data on biogas production rate. (HRT of 10 days, TS 3.5 % temperature cycle (a) 20 – 40°C and (b) 15

$$\gamma_v(t) = \gamma_o + \sum_{n=1}^{\infty}[a_n \cos(2\pi n\varphi t) + b_n \cos(2\pi n\varphi t)] \quad (16)$$

Where:
$\gamma_v(t)$ is the biogas production at a time t (L/L.d), γ_o is the average biogas production per unit volume of reactor (L/L.d), φ is the dominant frequency of the cycle = 1.22 x 10^{-5} (Hz) and a_n, b_n are Fourier coefficients.

The Fourier coefficients and mean daily biogas production rate can be obtained by regression analysis. These values will vary with operating conditions (cyclic temperature ranges, loading rates, etc)

Where the operating conditions (loading rate, pH, etc) are not favourable, the production follows a sinusoidal pattern which may be embedded in some harmonics and noise.

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